Abstract- Inverse Synthetic Aperture Radar (ISAR) imaging technique employs both wideband characteristics of radar waveform and the diversity of viewing angles from radar to a moving aerial target. The range resolution is proportional to the bandwidth of the waveform used, and the cross-range resolution is dependent on both the coherent processing interval and the target rotational motion as seen from different radar viewing angles. While the target’s relative movement with respect to the radar sensor provides the angular diversity required for Range-Doppler ISAR imagery, high-speed motion of space targets cause distortion problems like blurring caused by complex target motion. Trying to estimate motion parameters and to mitigate the undesired effects of motion on the ISAR image is often called motion compensation. The motion compensation (MOCOMP) process is one of the most challenging tasks in ISAR imaging research. One of the popular MOCOMP techniques to estimate the motion parameters and removing translational motion effects in ISAR images is called the minimum entropy method. Here we study the minimum entropy method for phase correction and show the effectiveness of this method to compensate motion effects and avoid image distortion by Matlab simulation results.

Keywords: Inverse synthetic aperture radar (ISAR), Motion compensation, Minimum entropy

1. INTRODUCTION

Inverse synthetic aperture radar (ISAR) has been proven to be a powerful signal processing tool for imaging moving targets. ISAR imagery plays an important role in applications such as target identification and classification. In situations where other radars display only a single unidentifiable bright moving pixel, the ISAR image is often adequate to discriminate between various missiles, military aircraft, and civilian aircraft.

To successfully form an ISAR image, the target’s motion should contain some degree of rotation with respect to the radar line of sight (RLOS), these may include rotational (yaw, roll, and pitch) motion parameters such as velocity, acceleration. The conventional way of obtaining a 2-D ISAR image is by collecting the frequency and aspect of diverse backscattered electromagnetic (EM) field data from the target. The ISAR image is constructed by Fourier-based signal processing, collecting the scattered field for different look angles and Doppler histories. However, in ISAR scenarios the target is often engaged in complicated maneuvers and the translational motion should be considered before performing imaging processing.

When the target is moving, the Doppler shift results from the scattering points on RLOS sets “inaccurate” distance information, due to the change in the phase of received EM wave. While the target is moving fast, it may occupy several pixels in the image during the integration interval of ISAR. Therefore, the phase of the backscattered wave is altered such that the resultant ISAR image is mislocated in cross range and blurred in both range and cross-range domains. If the target is not moving fast, the ISAR image may not be blurred. However, the location of the scatter points will still not be true due to the Doppler shift imposed by the target’s movement [1].
This report is organized as follows. In Section II the conventional procedure for obtaining an ISAR image is explained. Section III presents Doppler frequency shifts due to target motion, Section IV concentrates on the phase error induced by Doppler frequency shift, Section V focuses on motion compensation by formulating the minimum entropy for removing translational motion effects in ISAR images. In Section VI the effectiveness of the minimum entropy scheme for phase adjustment is investigated by Matlab simulations. Finally, Section VII presented conclusions of the report.

II. ISAR IMAGING BACKGROUND

The geometry of a radar imaging a target is shown in Figure 1. For simplicity, we only show a 2-D target with the radar located in the target plane containing the U-axis. At time $t$ the distance $r(t)$ from the radar antenna to the scattering point $(x, y)$ is approximated by

$$r(t) \approx R(t) + x \cdot \cos \theta(t) - y \cdot \sin \theta(t)$$

(1)

$R(t)$ is the distance from the radar to the rotation center of the target, and $\theta(t)$ is the rotation angle. Assuming the radar transmits a sinusoidal waveform with a carrier frequency $f$, the returned baseband signal from the scattering point becomes:

$$s(f, r) = \sigma(x, y) \exp(-j2\pi f \frac{2r}{c})$$

(2)

where $\sigma(x, y)$ denotes the complex reflectivity function of the scattering point and $c$ is the speed of the light. Therefore, the returned signal from the target can be represented as the integration of the returned signals from all scattering points in the target

$$S(f) = \iint_{-\infty}^{+\infty} \sigma(x, y) \exp\left(-j2\pi f \frac{2r}{c}\right) dx dy.$$  

(3)

Substituting (1) into (3), we have
\[ S(f) = \exp \left( -j4\pi f \frac{R(t)}{c} \right) \times \int_{-\infty}^{\infty} \sigma(x, y) \exp \left[ -j2\pi (xf_x - yf_y) \right] dx dy \quad (4) \]

where

\[ f_x = \frac{2f}{c} \cos \theta(t) ; \quad (5) \]

\[ f_y = \frac{2f}{c} \sin \theta(t) . \quad (6) \]

From (4) we see that if the target range \( R(t) \) is known over the entire observation time, the irrelevant phase term of the translational motion of the target can be removed by multiplying \( \exp \left( -j4\pi f \frac{R(t)}{c} \right) \) on both sides of (4). Therefore, the complex reflectivity density function \( \sigma(x, y) \) can be obtained by taking the inverse Fourier transform of the phase compensated frequency signal. The procedure of estimating \( R(t) \) and correcting the signal phase is called translational motion compensation [3].

To form a radar image, after measuring the returned signal in the I-Q channels, an M-by-N 2-D complex data matrix \( S(m, n) \) can be obtained, where \( m = 0, 1, \ldots, M -1; n = 0, 1, \ldots, N -1; \) and \( M \) is the number of the returned echoes, and \( N \) is the number of the samples of each echo in its frequency domain. This data matrix is regarded as the discrete form in the 2-D frequency domain of \( S(f) \) in (4). For simplicity, we assume the target rotation rate is a constant and the change of the aspect angle is very small. Rotational motion compensation and polar reformatting is not considered here. Then the range compression is performed as an N-point inverse discrete Fourier transform (IDFT) for each of the M received frequency signals, i.e.,

\[ G(m, n) = IDFTn[S(m, n)] \quad (7) \]

Where \( IDFTn \) denotes the inverse discrete Fourier transform operation with respect to the variable \( n \). Then, M range profiles, each containing N range cells, can be obtained. At this time, the scheme of translational motion compensation is carried out. The compensated range profiles become \( G'(m, n) \). Finally, the M-by-N image is obtained by

\[ I(m, n) = DFTm[G'(m, n)] \quad (8) \]

Where \( DFTm \) denotes the discrete Fourier transform operation with respect to the variable \( m \), and \( I(m, n) \) is called the radar image, which is the target’s reflections mapped into the range-Doppler plane.

III. DOPPLER EFFECT DUE TO TARGET MOTION

Real targets such as planes usually have complicated motion parameters while maneuvering. These parameters are unknown, therefore must be estimated and then undesired effects of motion on the ISAR image must be eliminated to have a successful ISAR image of the target. Here the effect of the target’s motion to the phase of the backscattered wave and/or to the ISAR image is investigated based on the geometry illustrated in Figure 1. The target may have both radial and rotational motion during the illumination period of the radar. For this reason, the scatter point
P (x, y) on the target is assumed to have both radial and rotational motion components. The phase center is selected in the middle of the target and is assumed to be the origin. We would like to estimate the phase error induced due to target motion. If the target is situated in the far field of the radar, the distance of point P (x, y) from the radar can be approximated by (1). Expanding $R(t)$ and $\theta(t)$ into their Taylor series, they can be represented as

$$R(t) = R_0 + v_t t + \frac{1}{2} a_t t^2 + ... \quad (9)$$
$$\theta(t) = \theta_0 + \omega_r t + \frac{1}{2} \alpha_r t^2 + ... \quad (10)$$

Here, $R_0$ is the initial range of the target, and $v_t$ and $a_t$ are the target’s translational velocity and the acceleration, respectively. Higher order terms starting from the target’s translational motion follow these first three terms. Similarly, $\theta_0$ is the initial angle of the target with respect to the RLOS axis. $\omega_r$ and $\alpha_r$ are the angular velocity and the angular acceleration of the target, respectively.

The phase of the backscattered signal from point P can be written as

$$\theta(t) = -j2\pi f \frac{2r(t)}{c} \quad (11)$$

Therefore, the Doppler frequency shift due to motion can be calculated by taking the time derivative of this phase as

$$f_D = \frac{1}{2\pi} \frac{\partial}{\partial t} \theta(t) = \frac{2f}{c} \frac{\partial}{\partial t} r(t)$$
$$= - \left( \frac{2f}{c} (v_t + a_t + \cdots) + \frac{2f}{c} (\omega_r + \alpha_r + \cdots) \right) x \sin \theta(t) + y \cos \theta(t). \quad (12)$$

Here, the first and second terms represent the radial (or translational) and the rotational Doppler frequency shifts, respectively.

**IV. RANGE MIGRATION AND PHASE ERROR IN ECHOED SIGNAL**

The received signal is given as the integration of the backscattered echoes from all the scattering points inside the radar beam, i.e.,

$$S(t) = \int_{-\infty}^{+\infty} A(x, y) \exp \left(-j4\pi f \frac{R(t)}{c} \right) dx dy. \quad (13)$$

where $A(x, y)$ is the backscattered signal intensity from any scattering point (x, y), and $f$ corresponds to the frequency of the radar waveform. Substituting the range equation in equation (1) into Equation (13), the received signal evaluates to

$$S(t) = \exp \left(-j4\pi f \frac{R(t)}{c} \right) \int_{-\infty}^{+\infty} A(x, y) \exp \left[\left(-j4\pi f \frac{c}{c} (x \cos \theta(t) - y \sin \theta(t)) \right) \right] dx dy. \quad (14)$$
If the target’s initial range, $R_0$, and the linear translational velocity, $v_t$, are known, the phase term in front of the above integral can be removed by multiplying (14) by $\exp\left(j4\pi f \frac{R(t)}{c}\right)$, this is called range tracking or coarse motion compensation and it is the standard procedure for compensating the translational motion effects after obtaining the phase compensated backscattered signal, a Fourier transform operation can then be applied to obtain image backscattered signal intensity function $A(x,y)$. However, image distortion/blurring is inevitable if only the range tracking procedure is used as the compensation tool. Assuming the radar transmits a chirp waveform

$$p_T(\tau) = \text{rect} \left( \frac{\tau}{T_p} \right) \exp \left( j\gamma \pi \tau^2 \right) \quad (15)$$

where $\tau$ denotes the fast time that controls the interval between the pulses, $T_p$ is the pulse duration, $\gamma$ is the chirp rate and $\text{rect} \left( \frac{\tau}{T_p} \right)$ is rectangular pulse centered at $\tau$ with duration of $T_p$. Therefore, the received signal from the target after down-conversion to base-Band is given by [4]

$$S_R(\tau, t) = \int_{-\infty}^{+\infty} A(x,y), p_T \left( \tau - \frac{2.r(t)}{c} \right). \exp \left( -j4\pi f \frac{r(t)}{c} \right) dx dy$$

(16)

By applying the Fourier transform with respect to $\tau$ and neglecting constants, we obtain

$$S_F(f_r, t) = \int_{-\infty}^{+\infty} A(x,y), p_T(f_r). \exp \left( -j4\pi \frac{r(t)}{c} (f_r + f_c) \right) dx dy$$

(17)

where $p_T(f_r) \approx \text{rect} \left( \frac{f_r}{T_p} \right) \exp\left( j\pi \frac{f_r^2}{T_p} \right)$ is the Fourier transform of $p_T(\tau)$. Applying the matched-filtering to $S_F(f_r, t)$ by multiplying with $p_T^*(f_r)$, the conjugate of $p_T(f_r)$, and omitting the introduced constant, the signal is expressed as

$$S_F(f_r, t) = \int_{-\infty}^{+\infty} A(x,y). \exp \left[ -j4\pi \frac{x \sin \theta(t) - y \cos \theta(t)}{c} (f_r + f_c) \right]. \exp \left[ -j4\pi \frac{R(t)}{c} (f_r + f_c) \right] dx dy.$$ 

(18)

Assuming the effective rotational velocity during the coherent processing interval (CPI) is $\omega_{eff}$ then we have the rotation angle $\theta(t) = \omega_{eff} \cdot t$. If we assume that during the small CPI for ISAR imaging, $\theta$ is very small, such as $\leq 3-4^\circ$. We can use the approximation $\cos \theta(t) \approx 1$ and $\sin \theta(t) \approx \omega_{eff} \cdot t$. Based on this approximation, we can rewrite (18) as

$$S_F(f_r, t) \approx \int_{-\infty}^{+\infty} A(x,y). \exp \left[ -j4\pi \frac{x \omega_{eff} \cdot t - y}{c} (f_r + f_c) \right] \exp \left[ -j4\pi \frac{R(t)}{c} (f_r + f_c) \right] dx dy.$$ 

(19)
In (19), the phase term \( \exp \left[ -j4\pi \frac{R(t)}{c} (f_r) \right] \) causes misalignment of range profiles in the range-compressed domain. And, the term \( \exp \left[ -j4\pi \frac{R(t)}{c} (f_c) \right] \) stands for the phase error. We can rewrite (19) as

\[
S_F(f_r, t) 
\approx \int_{-\infty}^{+\infty} A(x, y). \exp \left[ -j4\pi \frac{\omega_{eff} \cdot t}{\lambda} \right] . \exp \left[ j4\pi \frac{Y}{c} f_r \right] . \exp \left[ -j2\pi . \Delta t_e(t). f_r \right] . \exp \left[ -j\Delta \phi(t) \right] . dx dy.
\]

The range migration and phase error are denoted by \( \Delta t_e(t) = \frac{2R(t)}{c} \) (range shift) and \( \Delta \phi(t) = 2\pi \frac{2R(t)}{\lambda} \) (phase error), respectively. The phase in (20) is separated into four terms: the first term is the linear phase term corresponding to the Doppler, the second term corresponds to the range compression, the third term corresponds to the range migration, and the last term is the phase error from translational motion [4].

Considering the inevitable noise, the signal echoes can be expressed in a discrete form as

\[
S(m, n) = \hat{S}(m, n). \exp \left[ -j2\pi . \Delta t_e(n). m. \Delta f_r \right] . \exp \left[ -j\Delta \phi(n) \right] + \epsilon_F(m, n)
\]

where \( \epsilon_F(m, n) \) denotes the complex noise. Unless an optimal translational motion compensation is obtained serious blurring can result in the ISAR image formed by the inverse Fourier transform.

V. MINIMUM ENTROPY METHOD FOR MOTION COMPENSATION

Minimum entropy is a MOCOMP technique for removing translational motion effects in ISAR images. Entropy function is introduced to determine the degree of the alignment of the radar echoes. It assumes that if the echoes are accurately aligned, the shape of the summed envelope of the echoes will be sharpest, otherwise it will be smoothed or blurred. Entropy function is used to measure the degree of the sharpness [2]. In ISAR imaging, entropy can be used to measure the focus quality of an image. Better focus results in a sharper image and thus, smaller entropy.

The minimum entropy method tries to estimate the motion parameters (such as velocity and acceleration) of the target. This task is accomplished by calculating the entropy of the energy in the image and minimizing this parameter by iteratively trying out possible values of the motion parameters.

A. Minimum entropy for phase adjustment

In [5] phase adjustment is formulated as

\[
g(k, n) = \sum_{m=0}^{M-1} S(m, n). \exp[j\varphi(m)]. \exp\left( -j\frac{2\pi}{M} km \right)
\]
where \( g(k,n) \) is the complex image, \( S(m,n) \) is the signal resolved and aligned in range, and \( \varphi(m) \) is the adjustment phase. \( k, n \) and \( m \) are the indices of the Doppler frequency, range bins and pulses, respectively. In this equation, phase adjustment is first carried out by multiplying \( S(m,n) \) by \( \exp[j\varphi(m)] \), and azimuth imaging is then carried out by taking the Fourier transform of \( S(m,n).\exp[j\varphi(m)] \) with respect to \( m \). The key to phase adjustment is the estimation of \( \varphi(m) \).

In the minimum-entropy phase adjustment, \( \varphi(m) \) is determined to minimize the entropy of \( |g(k,n)|^2 \).

In the following, the principles of entropy will be explained. The entropy of \( |g(k,n)|^2 \) is defined as

\[
E[|g(k,n)|^2] = \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} \frac{|g(k,n)|^2}{S} \ln \frac{S}{|g(k,n)|^2}
\]  

(23)

where

\[
S = \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^2.
\]  

(24)

Equation (23) can be written as

\[
E[|g(k,n)|^2] = \ln S - \frac{1}{S} \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^2 \ln |g(k,n)|^2.
\]  

(25)

Since \( S \) is a constant, entropy can be redefined as

\[
E'[|g(k,n)|^2] = - \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} |g(k,n)|^2 \ln |g(k,n)|^2.
\]  

(26)

To solve this optimization, in general, minimum entropy phase adjustment is implemented by an iterative solution, the image entropy decreases with the increase of the iteration number until the estimate reaches an acceptable convergence value. But before solving this equation for minimum entropy we will describe the entropy equation for a more complex situation where we are looking to compensate both range and phase errors.

B. Joint range alignment and phase adjustment by minimum entropy

In practice, due to the complex motion of the target and the variance of the system and the environment, the translational motion between the target and the radar usually has high-order terms. For joint range alignment and phase adjustment by minimum entropy, the authors in [4] modeled the translational motion error as a K-order polynomial, i.e.,
\[ R(n) = \sum_{k=1}^{K} a_k(n.\Delta t)^k. \]  

(27)

the presence of the high-order terms in \( R(n) \) introduces both range migration and phase errors. Equation (21) can be written as

\[ S(m, n) = \tilde{S}(m, n). \exp \left[ -j4\pi \cdot \frac{R(n)}{c} \cdot (m.\Delta f_r + f_c) \right] + \varepsilon_F(m, n). \]  

(28)

By substituting (27) in to (28) we get

\[ S(m, n) = \tilde{S}(m, n). \exp \left[ -j4\pi \cdot \sum_{k=1}^{K} a_k(n.\Delta t)^k \cdot (m.\Delta f_r + f_c) \right] + \varepsilon_F(m, n). \]  

(29)

A polynomial coefficient vector can be defined as \( \alpha = [\alpha_1, ..., \alpha_k]_{1\times K} \) and gives the complex image after error correction by \( \tilde{\alpha} = [\tilde{\alpha}_1, ..., \tilde{\alpha}_k]_{1\times K} \) as follows

\[ g(k, h; \tilde{\alpha}) = \frac{1}{NM} \sum_{n=0}^{N-1} \exp \left[ j2\pi \cdot \frac{hn}{N} \right] \cdot \sum_{m=0}^{M-1} \exp \left[ j2\pi \cdot \frac{km}{M} \right] \]

\[ \times s(m, n). \exp[j4\pi \cdot \frac{\sum_{k=1}^{K} \tilde{\alpha}_k(n.\Delta t)^k}{c} \cdot (m.\Delta f_r + f_c)]. \]  

(30)

Motion correction in (30) is performed on the phase of \( s(m, n) \). Therefore, the entropy of image is defined as function of \( \tilde{\alpha} \), which is given by

\[ E_g(\tilde{\alpha}) = -\frac{1}{S_g} \sum_{k=0}^{M-1} \sum_{h=0}^{N-1} |g(k, h; \tilde{\alpha})|^2 \cdot \ln |g(k, h; \tilde{\alpha})|^2 + \ln S_g \]  

(31)

where the image intensity is rewritten as

\[ S_g = \sum_{k=0}^{M-1} \sum_{h=0}^{N-1} |g(k, h; \tilde{\alpha})|^2 \]  

(32)

The estimate of \( \alpha \) is obtained by minimizing the image entropy, expressed as

\[ \langle \tilde{\alpha} \rangle = \arg \min_{\tilde{\alpha}} E_g(\tilde{\alpha}) \]
C. Iterative approach for minimum entropy phase adjustment

From section A we found that adjustment phase must be found in such way that minimizes the entropy of the image in (25). In this section we are looking for $\phi(m)$ that minimizes $E'[|g(k, n)|^2]$, as described before such that

$$\frac{\partial E'[|g(k, n)|^2]}{\partial \phi(m)} = 0.$$  \hspace{1cm} (34)

In [6] the derivative of $E'[|g(k, n)|^2]$ with respect to $\phi(m)$ is obtained from (26) as

$$\frac{\partial E'[|g(k, n)|^2]}{\partial \phi(m)} = - \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} [1 + \ln|g(k, n)|^2] \frac{\partial |g(k, n)|^2}{\partial \phi(m)}$$  \hspace{1cm} (35)

Since $|g(k, n)|^2 = g(k, n) g^*(k, n)$

$$\frac{\partial |g(k, n)|^2}{\partial \phi(m)} = 2. Re\left[g^*(k, n) \frac{\partial g(k, n)}{\partial \phi(m)}\right].$$  \hspace{1cm} (36)

Substituting (37) into (36), one obtains

$$\frac{\partial E'[|g(k, n)|^2]}{\partial \phi(m)} = -2. Re\left\{- \sum_{k=0}^{M-1} \sum_{n=0}^{N-1} [1 + \ln|g(k, n)|^2] g^*(k, n) \frac{\partial g(k, n)}{\partial \phi(m)}\right\}. $$  \hspace{1cm} (37)

Substituting $m'$ for $m$ in (22), we have

$$g(k, n) = \sum_{m'=0}^{M-1} s(m', n). \exp[j\phi(m')]. \exp \left(-j \frac{2\pi}{M} km' \right).$$  \hspace{1cm} (38)

The derivative of $g(k, n)$ with respect to $\phi(m')$ is obtained from (38) as

$$\frac{\partial g(k, n)}{\partial \phi(m)} = j.S(m, n). \exp \left[j \phi(m) \exp \left(-j \frac{2\pi}{M} km \right)\right].$$  \hspace{1cm} (39)
In order to obtain (39), we notice that \( m \) is only a sample of \( m' \) here and

\[
\frac{\partial E'[|g(k,n)|^2]}{\partial \varphi(m)} = 2M \text{Im}[\exp[j\varphi(m)]a^*(m)]
\]

(40)

where

\[
a(m) = \sum_{n=0}^{M-1} s^*(m',n) \frac{1}{M} \sum_{k=0}^{m-1} \left[ 1 + \ln|g(k,n)|^2 \right] g(k,n) \exp \left( j \frac{2\pi}{M} km \right).
\]

(41)

Substituting (39) into (34), one obtains

\[
\varphi(m) = \angle a(m)
\]

(42)

Where \( \angle a(m) \) denotes the phase of \( a(m) \).

VI. SIMULATION RESULTS

In this section simulation results of ISAR image reconstruction will be demonstrated. The motion compensation method discussed above using minimum entropy for phase adjustment will be investigated to verify its effectiveness. The target is assumed to consist of a set of perfect scattering points that have equal scattering amplitudes. A 3D CAD model of a F-14 Tomcat as shown in Figure 2 was used as a target, a CAD to MAT file converter [7] is used to convert 3D CAD models to a Matlab Mat files. The aircraft is assumed to roll around its center.

![Figure 2 F14 Jet 3D CAT Model.](image)
Assuming sharp edges have more Radar Cross Section (RCS), we considered edges as scattering points, resulting from derivation of the cross section of the 3D model while maneuvering. Figure 3 depicts the scattering points of the target at one moment in time.

Figure 3 perfect scattering points of target.

The target, at an initial radial distance of $R_0 = 5$ km, is moving away from the radar with a radial velocity of $v_t = 4$ m/s. The target has a radial acceleration value of $a_t = 0.5$ m/s$^2$. We also assume that the target is rolling with an angular velocity of $\phi_r = 0.05$ rad/s.

The radar sends 128 bursts, each having 128 modulated pulses. The frequency of the first pulse is $f = 10$ GHz and the total frequency bandwidth is $B = 1$ GHz. The pulse repetition frequency (PRF) of the radar system is chosen as 40 KHz and the initial angle of target with respect to the radar is 55 degrees.

First, we obtained the conventional Range-Doppler ISAR image of the target by employing traditional ISAR imaging procedures [1] without applying any compensation for the motion of the target. The resultant raw range-Doppler ISAR is depicted in Figure 4.
As it can be clearly seen from the Figure 4, the effect of the target’s motion is severe in the resultant ISAR image such that the image is broadly blurred in the range and Doppler domains, and the true locations of the target’s scattering centers cannot be retrieved from the image. The uncompensated ISAR image is highly distorted and blurred due to both the translational and the rotational motion of the target.

In Figure 5, the spectrogram of the received time-pulses is plotted. This Figure demonstrates the progressive shift in the frequency (or in the phase) of the consecutive received time pulses. This shift occurs due to the change of the target’s range from the radar during the integration time of the ISAR process. If a successful MOCOMP practice is applied, there is expected to be no (or minimal) range shift between consecutive time pulses.
The minimum entropy methodology [8] is applied to the ISAR image data in Figure 4. The algorithm searches for the values of \( v_t \) and \( a_t \) by minimizing the entropy of the compensated ISAR image

\[
I' = F_2^{-1}\{S, E^s\}
\]  

(43)

where \( S \) is defined in (12) and (13) for different values of \( v_t \) and \( a_t \). The estimated values for \( v_t \) and \( a_t \) are found by iteratively searching the minimum value of the entropy [6] as defined in (26). Figure 6 shows the graph of the entropy value for different values of translational velocity and acceleration.

The 2D search space assumes a minimum value where \( v_t^{\text{est}} \) parameter becomes equal to 4 m/s and the \( a_t^{\text{est}} \) parameter equals 0.5 m/s\(^2\) as demonstrated in Figure 6. Therefore, the algorithm successfully estimates the correct values of \( v_t \) and \( a_t \).

![Figure 6 Entropy plot for translational radial velocity and translational radial acceleration search space.](image)

The effect of motion in the scattered field can then be mitigated by multiplying it by the compensating phase term as given in (22). Consequently, the motion-compensated ISAR image is obtained as shown in Figure 7 by applying the regular FFT-based ISAR imaging technique. The compensated ISAR image clearly demonstrates that the unwanted effects due to target’s motion are eliminated after applying the minimum entropy methodology. The target’s scattering centers are displayed with good resolution.
Figure 7 ISAR image of the maneuvered F-14 after applying minimum entropy compensation.

A further check is done by looking at the spectrogram of the motion-compensated received time pulses as illustrated in Figure 8. As it is obvious from this spectrogram, the range delays between the time pulses are eliminated such that all frequency (or the phase) values of the returned pulses are aligned successfully.

Figure 8 Spectrogram of range after compensation.

VII. CONCLUSION

In this report the mathematical description and original interpretation of ISAR signal formation and imaging is described, then two approaches of complex joint range alignment and phase adjustment are expressed. Phase adjustment is achieved by solving the equation of image entropy
minimization. It has been underlined that the image reconstruction is a procedure of motion compensation, applied to all phases induced by the scattering points at the moment of imaging. We focused on the minimum entropy technique for phase adjustment. As it is demonstrated in simulations, this technique is effective and efficient for phase adjustment in ISAR imaging. One of the drawbacks of this motion compensation technique is that multiple local minimums in the entropy can occur when the target is maneuvering fast. To find out a global minimum in the entropy and optimal values of the polynomial coefficients the computation process has to be enlarged to a wider interval. The subject of future research will investigate efficiency and effectiveness of joint range alignment and phase adjustment by minimum entropy with higher order terms of the phase correction.

REFERENCES


